NOTATION

τ, Reynolds frictional stress; $\langle \rangle$, averaging in time; ρ, density; x, longitudinal coordinate; y, transverse coordinate; u', longitudinal fluctuation velocity; v', transverse fluctuation velocity; u, average longitudinal velocity; L_x, path length of longitudinal relaxation; μ_t, eddy dynamic viscosity; $\varkappa$, K, a, empirical constants; x', x", integration variables in relation (2); subscript 0 refers to the value of a quantity in the initial section; u_e, average velocity at the outer edge of a boundary layer; δ^*, displacement thickness; δ^{**}, momentum thickness; and δ, local thickness of a boundary layer.

LITERATURE CITED

1. J. O. Hinze, "Memory effects in turbulence," Angew. Math. Mech., 56, 403-415 (1976).
2. P. J. H. Builtjes, "Memory effects in turbulent flows," WTHD, 97, Tech. Hogeschool, Delft Technological University (1977), Preprint of Doctoral Thesis.
3. L. G. Loitsyanskii, "Aftereffect phenomena in turbulent motion," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 5-19 (1982).
4. F. Klauser, "Turbulent boundary layer," in: Problems of Mechanics [Russian translation], Vol. 2, IL, Moscow (1959), pp. 297-340.
5. J. Tsuij and J. Morikawa, "Turbulent boundary layer with pressure gradient alternating in sign," Aeronaut. Q., 27, Part 1, 15-28 (1976).
6. T. Sebesy and A. M. Smith, "Finite-difference method of calculating compressible laminar and turbulent boundary layers," Teor. Osnovy Inzh. Raschetov, No. 3, 121-133 (1970).

CORRELATION BETWEEN AMPLITUDES OF HARMONIC COMPONENTS OF VELOCITY PULSATIONS

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UDC 532.517.4

A correlation is empirically established between the amplitudes of harmonic components of velocity pulsations of a turbulent flow next to a wall.

In analyzing histograms of velocity pulsations for turbulent steady-state flow, the flow regime is often modeled as a steady, stochastic process. This makes it possible to represent it as a set of elemental harmonic components. Here, it is assumed that the harmonic components are not coupled to each other (do not correlate), so that, within the framework of this model, it is sufficient to find the dispersion of the components (spectra). The sum of these components, meanwhile, equals the power of the process.

In studying the mechanism by which energy is transferred from one perturbation to another, it is of interest to know not only the dispersion of the components, but also the parameters of their interaction. Here, we note that the possibility of a connection existing between the components is probabilistic rather than rigorous (determined), since energy is divided and transferred at random moments of time and the division takes place on structures with random dimensions.

As concerns correlations between harmonic components, it is necessary to regard the process as unsteady, i.e., its characteristics will depend on time, and the sum of the dispersions of the components (spectra) will not be equal to the power of the process, since part of this power is spent on the interaction between the components. The periods of transience which occur due to the correlation between the components may be comparable to the periods of these components, so that they cannot be detected by the time-averaging methods of analysis which are widely used.

Harmonizable processes [1-3], which can be grouped into several classes, may serve as a model of a signal representing velocity pulsations in steady flow which will allow for a correlation between the (harmonic) components. Dragan introduced the class of D-harmonizable processes, the criterion of which is a finite total dispersion of the components. This condition is satisfied for velocity histograms.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 44, No. 4, pp. 564-567, April, 1983. Original article submitted November 10, 1981.

0022-0841/83/4404-0379$07.50

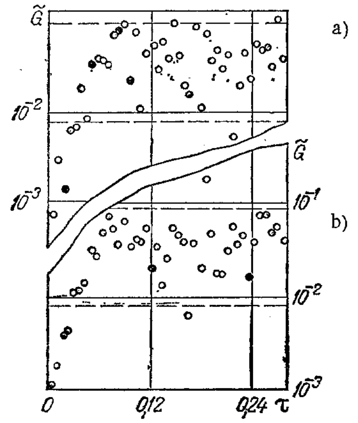

Fig. 1. Dependence of the spectral density of the amplitudes $\tilde{G}$ ($m^2 \cdot sec^{-3}$) on time τ (sec) (the dashed line is the boundary of the region beyond which 95% confidence intervals do not cover the arithmetic mean); interval of the recorded velocity histograms: a) at the beginning of the recording; b) at the end.

Since harmonics are characterized by amplitude, frequency, and phase of oscillation, more complete study of the correlation properties of harmonics requires investigation of the correlation between amplitudes and phases at different frequencies, as well as their mutual correlation.

We developed an algorithm and experimentally checked the correlation between the amplitudes of the harmonic components of velocity in the case of steady turbulent flow in a circular pipe. The amplitudes $A(f_i)$ can be evaluated by the sum

$$\hat{A}(f_i)=M\hat{A}(f_i)+\xi(f_i),\ f_i=i/N\Delta t,\ i=0,\ 1,\ 2,\ \ldots,\ N-1.$$

The spectrum of the function $A(f_i)$ is equal to $\tilde{G}(\tau_k) = G(\tau_k) + G_\xi(\tau_k)$, $k = 0, 1, 2, \ldots, N-1$, since $M\xi(f_i) = 0$. The parameter τ_k has the dimension of time.

For a steady process of the "white" noise type (no correlation between the harmonics), $G(\tau_k) = A_0\delta(\tau_k)$, $G_\xi(\tau_k) = \text{const}$. Thus, for "white" noise type processes, nonsatisfaction of the relation

$$\tilde{G}(\tau_k)=\text{const},\ k=1,\ 2,\ \ldots,\ N-1 \tag{1}$$

indicates the correlation of the function $\xi(f_i)$, and, thus, of the amplitudes at different frequencies. In the case of a varying amplitude spectrum, violation of condition (I) may be due to the $G(\tau_k)$ term. It is therefore eliminated by changing over to a process with a constant spectrum. This operation does not alter the correlation properties of the function $\xi(f_i)$, and Eq. (1) will be satisfied for the transformed process in the case of noncorrelation of the amplitudes.

A program was written for the "Dnepr-21" computer as an algorithm to check for correlation. The computing sequence is as follows:

1. The values of the amplitudes are determined

$$\hat{A}(f_i)=\text{Re}^2 F[x(t_i)]/N\Delta t,$$

where $F[x(t_i)]$ is the Fourier transform, found from a sample of the signal $x(t_i)$ using a rapid Fourier transform algorithm.

2. The amplitude of the process, converted to a process with a constant spectrum, is found:

$$A_H(f_i)=\hat{A}(f_i)\Bigg/\frac{1}{2\varepsilon+1}\sum_{k=i-\varepsilon}^{i+\varepsilon}\hat{A}(f_k)\quad i=(\varepsilon+1),\ \ldots,\ N-1-\varepsilon,$$

where $2\varepsilon + 1$ is a parameter, the choice of which depends on the type of spectrum characterizing the process. In the present case, it is equal to 11.

3. A sample of the function $A_H(f_i)$ is subjected to spectral analysis. We averaged values of the periodograms at three adjacent frequencies, which corresponds to six degrees of freedom.

If the amplitudes do not correlate, then we must obtain random values with a constant mathematical expectation. The arithmetic mean of the computed estimates can be taken as an estimate of this constant. If the estimates actually satisfy the hypothesis of constant $\tilde{G}(\tau_k)$, then the confidence intervals [4], constructed relative to the computed values with a confidence level, should cover their arithmetic mean. If the relative number of points outside this region is greater than the confidence level, then the hypothesis of noncorrelation of the amplitudes of the harmonics is invalid.

To detect a correlation among velocity pulsation amplitudes, it is necessary to create a turbulent flow such that eddies are formed regularly and the division process is impeded. Thus, the number of eddies formed will not be so great as to hinder detection of their interaction. We therefore chose to study the motion of a clay solution in a circular pipe. The turbulence mechanism remains the same in this case, while division is impeded by the action of cohesive forces.

The generation and development of turbulence in a clay solution in a circular pipe was studied in [5], where the experimental unit is described. The measurements were made in the region of developed turbulence at a flow-rate velocity twice as great as the flow-rate velocity at which the alternation coefficient reached unity throughout the core of a clay flow with a concentration of 7.5%. The flow-rate velocity was equal to 2.2 m/sec. The experiment was conducted on a closed-type hydraulic unit with a measurement section $9.8 \cdot 10^{-2}$ m in diameter. The velocities were measured with a conduction anemometer with a constant external magnetic field at a distance of 50 diameters from the inlet. The amplified signal from the primary transducer was recorded on an M-168 tape recorder and the signal from the recorder was analyzed on a "Dnepr-2" computer. The size of the data sample N = 512, which corresponds to a realization time of 1.1 sec.

Figure 1 shows results of the calculations for the transverse velocity component at the wall for the histogram interval at the beginning (a) and end (b) of the record. It is apparent from the figure that the hypothesis of constant $\tilde{G}(\tau_k)$ is not satisfied, since 14 and 12% of the points lie beyond the indicated region. It should be emphasized that, after $\tau = 0.052$, the points are distributed around a straight line, and there is no transience in this period interval.

NOTATION

$\hat{A}(f_i)$, amplitude of elementary harmonic components; f, frequency; N, data sample; Δt, digitization time; M, mathematical expectation; $G_\xi(\tau_k)$, spectrum of the function $\xi(f_i)$; $\xi(f_i)$, deviation function; $G(\tau_k)$, spectrum of the amplitude spectrum; $\delta(\tau)$, Dirac delta function; $F[x(t_i)]$, Fourier transform of the function $x(t_i)$; τ, time.

LITERATURE CITED

1. M. Love, Probability Theory [Russian translation], IL, Moscow (1962).
2. Yu. A. Rozanov, "Spectral analysis of abstract functions," Probability Theory and Its Application, 4, No. 3, 291-310 (1959).
3. Ya. P. Dragan, "Random processes with a finite mean power, their spectra and harmonicity," Summary of Documents of the Second Vilnius Conference on Probability Theory and Mathematical Statistics, Izd. Akad. Nauk Lithuanian SSR, Vilnius (1977), pp. 133-134.
4. G. M. Jenkins and D. G. Watts, Spectral Analysis and Its Applications, Holden-Day (1968).
5. I. L. Povkh, N. I. Bolonov, and A. E. Éidel'man, "Generation and development of turbulence in the motion of a disperse system in a circular pipe," Inzh.-Fiz. Zh., 26, No. 5, 901-907 (1974).